

- Initial assumption same as jet: 'rate at which weight is reduced = rate at which fuel burnt

- However for propeller engine, f = mass of fuel burnt per unit of power per second

rather than thrust for jet

$$\frac{dW}{dt} = -fgp$$

f for prop different to f for jet.

Power delivered by propeller, $P = \frac{DV}{\eta}$ ← propeller efficiency

$$\frac{dW}{dt} = -fg \frac{DV}{\eta} \rightarrow \int dt = -\frac{\eta}{fg} \frac{C_L}{C_D} \int \frac{dW}{W}$$

$$E_{prop} = t_{12} = \frac{\eta}{fg} \frac{1}{v} \frac{C_L}{C_D} \ln\left(\frac{W_1}{W_2}\right)$$

For max endurance we require max

$$\frac{1}{v} \frac{C_L}{C_D}$$

or minimise

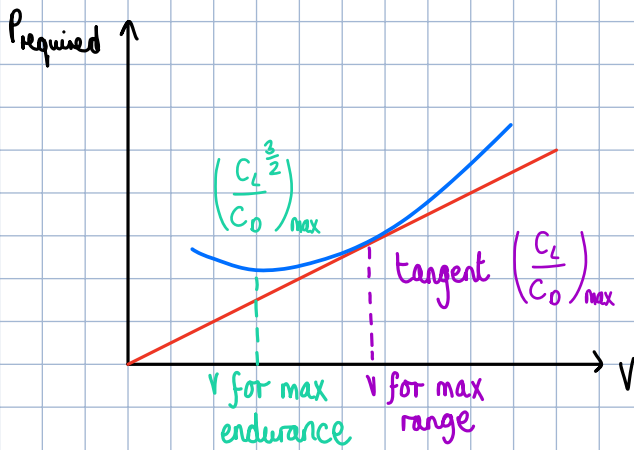
$$\frac{v C_D}{C_L} = \frac{v D}{W}$$

↳ maximise $\frac{C_L^{3/2}}{C_D}$

DV is power required to overcome drag
→ max endurance at min power speed

$$R_{prop} = S_{12} = \frac{\eta}{fg} \left(\frac{C_L}{C_D}\right) \ln\left(\frac{W_1}{W_2}\right) \quad (E_{prop} \times v)$$

max range at minimum drag speed



Drag Polar Relationships :

$\frac{C_L}{C_D}$ relation	Maximised When	C_L	C_D	Relates to
$\frac{C_L}{C_D}^{3/2}$	$C_{D_0} = \frac{1}{3} K C_L^2$	$\sqrt{\frac{3C_{D_0}}{K}}$	$4C_{D_0}$	Min power required Min sink rate Max prop endurance
$\frac{C_L}{C_D}$	$C_{D_0} = K C_L^2$	$\sqrt{\frac{C_{D_0}}{K}}$	$2C_{D_0}$	Min drag Max glide angle Max prop range Max jet endurance
$\frac{C_L^{1/2}}{C_D}$	$C_{D_0} = 3K C_L^2$	$\sqrt{\frac{C_{D_0}}{3K}}$	$\frac{4}{3} C_{D_0}$	Max jet range

Powers of C_L change according to velocity term in equation
 $\rightarrow V$ includes $\frac{1}{\sqrt{C_L}}$ term \therefore if multiplied by $V \rightarrow C_L^{1/2}$
 divided by $V \rightarrow C_L^{3/2}$
 V not included $\rightarrow C_L$

Payload-Range Diagrams :

- Represents trade off between aircraft payload and range
 \hookrightarrow provides understanding of utility of airplane

Aircraft Weight :

Max. take-off weight (MTOW) - max authorized weight for takeoff due to strength & airworthiness requirements

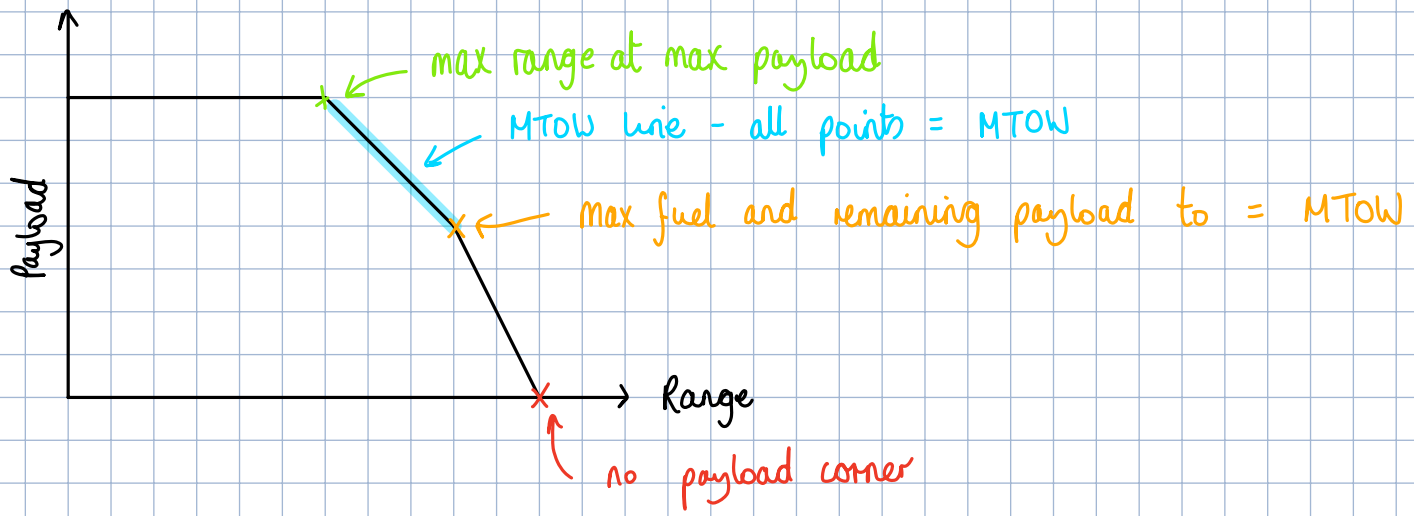
Operational empty weight (OEW) - basic weight of aircraft including all equipment and supplies needed for standard operation (does not include fuel & payload)

Max zero-fuel weight (MZFW) - max weight allowed before fuel is loaded

$$W_{\text{final}} = \text{OEW} + \text{payload} + \text{reserve fuel weight}$$

$$W_{\text{initial}} = W_{\text{final}} + \text{trip fuel weight}$$

Example Diagram:



Aircraft families (e.g. A350-900 vs 1000) made to have different payload-range lines.